Modelling Bioheat Transport in the Lung

An Application to Bronchial Thermoplasty

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Motivation

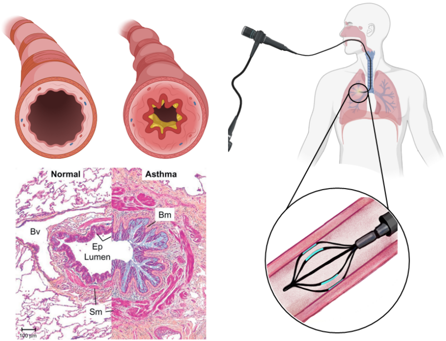


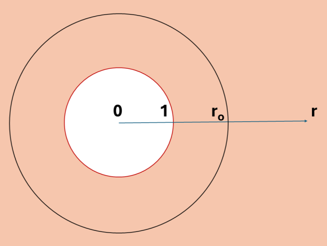
Figure: Credit - Biorender, Nadel (2016), Chernyavsky (2016).

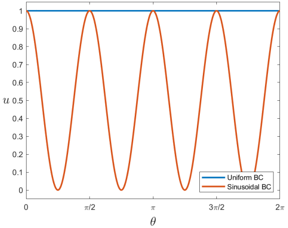
- Asthma is a heterogenous chronic inflammatory disease of the airway.
- It is commonly characterised by an increase in airway smooth muscle (ASM).
- Bronchial thermoplasty (BT) is a treatment for severe asthma.
- It heats the airway wall interior via four electrodes.
- This is believed to eliminate some of the ASM cells.
- Why do we need to model it?

Aim

- Explore some simple models of heat transfer during the procedure, and see what insight this can provide.
- Suggest some more novel and unexplored modelling methods that could be implemented in an attempt to uncover phenomena unobserved to date.

Steady state heat diffusion





Equation system:

$$\nabla^2 u = 0,$$
$$u(1,\theta) = \cos^2(2\theta),$$
$$u(R_o,\theta) = 0.$$

Dimensionless variables:

$$u = \frac{u^* - u^*_{\text{body}}}{u^*_0},$$
$$r = \frac{r^*}{r^*_{\text{i}}}.$$

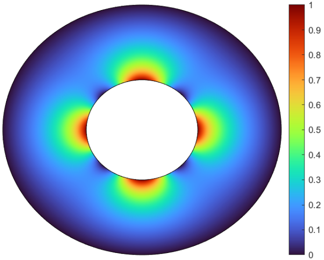
Sinusoidal boundary condition

Expanding the internal boundary condition

$$u(1,\theta) = \cos^2(2\theta) = \frac{1}{2}(\cos(4\theta) + 1),$$

we arrive at the result

$$u(r,\theta) = \frac{1}{2}\left(1 - \frac{\log(r)}{\log(R_o)} + \frac{1}{1-(R_o)^8}\left(r^4 - (R_o)^8 r^{-4}\right)\cos(4\theta)\right).$$



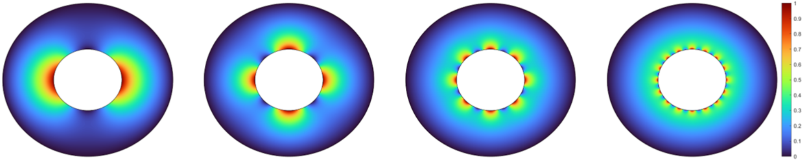
Varying the number of electrodes

Adjusting for a general number of electrodes N, the boundary condition becomes

$$u(1, \theta; N) = \cos^2\left(\left(\frac{N}{2}\right)\theta\right) = \frac{1}{2}(\cos(N\theta) + 1),$$

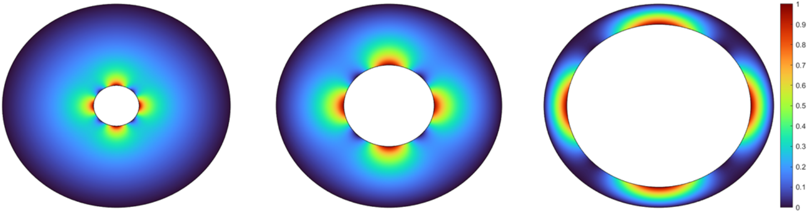
and we arrive at the result

$$u(r, \theta; N) = \frac{1}{2}\left(1 - \frac{\log(r)}{\log(R_o)} + \frac{1}{1 - (R_o)^{2N}}\left(r^N - (R_o)^{2N} r^{-N}\right)\cos(N\theta)\right).$$



Varying the size of the annular domain

We can see how doubling or halving our annular radius R_o would affect our solutions.

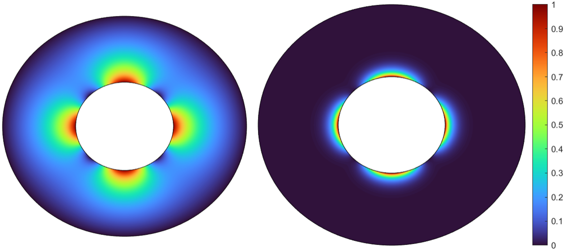


Including dissipation effects

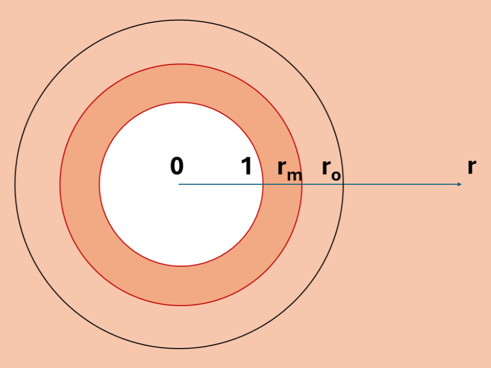
We can adjust the governing equation itself by adding a dissipation term,

$$\nabla^2 u - ku = 0.$$

This leads to a similar result to that achieved earlier, using modified Bessel functions in placed of our powers of r.



Layering our continuum



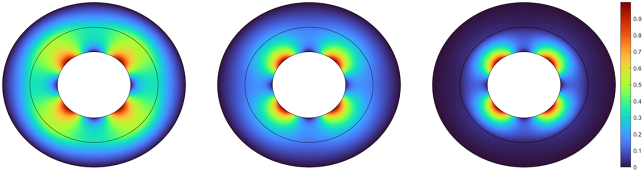
- We divide our continuum into two ‘layers’, 1 and 2, with differing thermal conductivities κ_1 and κ_2.
- The additional interface conditions are that the fluxes must be conserved at the boundary and the temperature distributions must be continuous at the boundary.

Layering our continuum

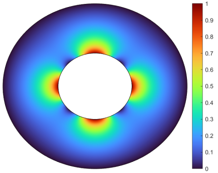
Our solution is very similar to that for one layer, taking the form

$$u_i(r,\theta) = A_i + B_i \log r + (C_i r^4 + D_i r^{-4})\cos(4\theta),$$

where A_i, B_i, C_i and D_i are constants dependent on the relative conductivity between the two layers (κ_1/κ_2) and the relative sizes the two layers.



Summary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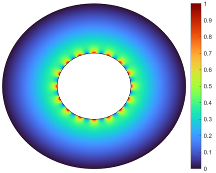
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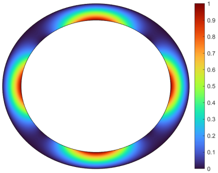
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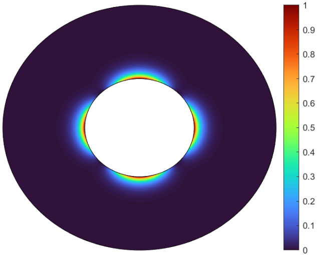
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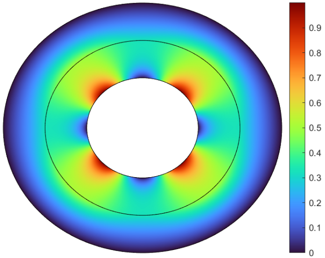
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Where can we go from here?

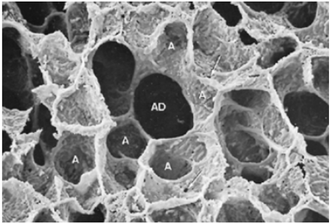

Figure: Credit: Weibel (2009).

Some potential next steps could include (but are not limited to):

- Considering how the microstructure of the lung could affect heat transport, via multi-scale homogenisation.
- Modelling how the tissue cools after heating.
- Considering the effects of other components of the airway, such as the mucus.
- Investigating effects of uncertainty in the parameters chosen (such as the conductivity).